Numerical Differentiation/Integration

Example 1

The following data is used in question 1:

x	0	1	2	3	4	5
y	5.68	6.75	7.32	7.35	6.88	6.24

1.a. Calculate the 1st order derivatives

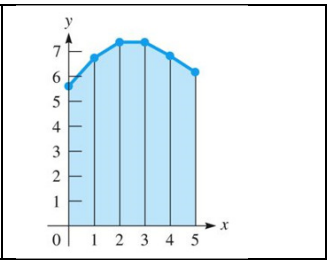
x	0	1	2	3	4	5
y	5.68	6.75	7.32	7.35	6.88	6.24
dy/dx	-	0.82	0.3	-0.22	-0.555	-

1.b. Calculate the 2nd order derivatives

x	0	1	2	3	4	5
y	5.68	6.75	7.32	7.35	6.88	6.24
d^2y/dx^2	-	-0.5	-0.54	-0.5	-0.17	-

1.c. Use the trapezoidal method to calculate the integral of the data between x = 0 and x = 4.

Plotted, the data looks like this, with a step size Δx = 1.



Trapezoid integral = 1.0 (5.68/2 + 6.75 + 7.32 + 7.35 + 6.88/2) = 27.70

1.c. Use Simpson's 1/3 rule to calculate the integral of the data between x = 0 and x = 4.
Integral by Simpson's rule = 1/3(5.68 + 4(6.75) + 2(7.32) + 4(7.35) + 6.88)

= 1.0/3(5.68 + 27.0 + 14.64+ 29.4 + 6.88) = 27.87

Example 2

Calculate the integral of the following data using both the trapezoidal rule and Simpson's 1/3 rule

x	2	4	6	8	10	12	14
y	0.670	2.34	4.56	3.67	3.56	4.78	6.87

The step size is Δx = 2.

Answer. Using the trapezoidal rule
Integral = 2(0.670/2 + 2.34 + 4.56 + 3.67 + 3.56 + 4.78 + 6.87/2) = 45.36

Answer. Using Simpson's 1/3 rule
Integral = 2/3(0.67 +4*2.34 + 2*4.56 + 4*3.67 + 2*3.56 + 4*4.78 + 6.87)
= 2/3(0.67 + 9.36 + 9.12 + 14.68 + 7.12 + 19.12 + 6.87) = 44.63

Example 3 – Approximating an integral of a mathematical function

(If you can't integrate a function analytically, you can evaluate the function at regular steps and then integrate it numerically.)

Using both the trapezoidal method and Simpson's 1/3 method, evaluate the integral of the function

f(x) = $x\sqrt{(x+1)}$

over the range x = 2.0 to x = 3.0 in steps of 0.1

(the exact integral = 4.6954 to 4 decimal places.)

Answer

By evaluating the function, the data for the integral is

x	2.0	2.1	2.2	2.3	2.4
f(x)	3.4641016	3.6974315	3.9354796	4.1781575	4.4253813

x	2.5	2.6	2.7	2.8	2.9	3.0
f(x)	4.6770717	4.9331532	5.1935537	5.4582048	5.7270411	6.0

a. Trapezoidal

Integral = $\frac{0.1}{2}[3.464\,101\,6 + 2(3.697\,431\,5) + \cdots + 2(5.727\,041\,1) + 6.000\,000\,0]$

= 4.6958 to 4 decimal places

b. Simpson 1/3

$$\int_2^3 x\sqrt{x+1}\,dx = \frac{0.1}{3}\Big[3.464\,101\,6 + 4(3.697\,431\,5) + 2(3.935\,479\,6) + 4(4.178\,157\,5) + 2(4.425\,381\,3) + 4(4.677\,071\,7) + 2(4.933\,153\,2) + 4(5.193\,553\,7) + 2(5.458\,204\,8) + 4(5.727\,041\,1) + 6.000\,000\,0\Big]$$

$$= \frac{0.1}{3}(140.86156) = 4.695\,385\,4$$

= 4.6954 to 4 decimal places

which agrees with the exact value to this number of decimal places

Example 4

The velocity v (kilometres per hour) of a car was recorded at 1 minute intervals as shown:

t(min)	0	1	2	3	4	5	6	7	8	9	10
V(km/h)	60	62	65	69	72	74	76	77	77	75	76

4.a. Calculate the acceleration of the car in kilometres/hour/hour

Answer. The acceleration is the 1st derivative of the velocity.

The step size between measurements Δx = 1 minute = 1/60 hour

t(min)	0	1	2	3	4	5	6	7	8	9	10
V(km/h)	60	62	65	69	72	74	76	77	77	75	76
Accn(km/h/h)	-	150	210	210	150	120	90	30	-60	-30	-

4.b. Calculate the distance travelled over the 0 to 10 minutes using the trapezoidal method
Answer
Distance = 1/60(60/2 + 62 + 65 + 69 + 72 + 74 + 76 + 77 + 77 + 75 + 76/2) = 11.917 km

4.c. Calculate the distance travelled over the 0 to 10 minutes using Simpson's 1/3 method.
Answer
Distance = (1/60)/3(60 + 4(62) + 2(65) + 4(69) + 2(72) + 4(74) + 2(76) + 4(77) + 2(77) + 4(75) + 76)
= 11.911