Last Name: _____

First Name: _____

Student Number: _____

Instructions:

1. Print your name and student number above AND on the MC answer sheet. A test or MC answer sheet without a name and student number won't be marked. A page (except the front page) without a name at the top right, where shown, won't be marked.
2. Use only a pencil when filling in the MC answer sheet for the multiple choice questions. Circle the correct answers on your question paper first and only when you are certain of your answer fill in the MC answer sheet. Only the answers found on the MC answer sheet will be used when marking the multiple choice questions.
3. Check that you have all 4 PAGES before beginning the exam.
4. Pace yourself – you have ~45 minutes.
5. Use the blank spaces on exam pages for rough work. No scrap paper is permitted.
6. Simple non-programmable calculators (not cell phones or tablets) are allowed – but you can leave your answers as accurate and complete numerical expressions where calculations are required but need not waste time evaluating them.
7. If you have a cell phone or any electronic device (other than a pacemaker) with you – be sure it is turned off now, and stored in a safe place away from your desk.
8. Hand in BOTH this exam booklet AND the MC answer sheet. Taking an exam booklet from the exam room will result in an automatic grade of 'F' for this course.
9. This test is worth 17.5% of your final mark.

I have read, understood, and will comply with all of the above instructions:

_____ _____

sign your full name here date

Formulas are at the end. Choose the **best** answer in each of the following multiple-choice questions**.**
1 mark each

1. For a `float` data type the hidden bit has a value
a. 2^0 b. 2^1 c. 2^{-23} d. 2^{127} e. 2^{255}

2. A `float` number can be written as $m.10^n$ where m is between 1 and 10 and n is in the range
a. -7 to +7 b. 0 to 127 c. 0 to -23 d. -38 to +38 e. -128 to +127

3. The ***mantissa*** of a float data type **in memory** consists of bits representing values in the range
a. 2^0 to 2^{-23} b. 2^{-0} to 2^{-23} c. 2^{-1} to 2^{-23} d. 2^{-0} to 2^{-255} e. 2^{-128} to 2^{+127}

4. When two float numbers are added
a. the smaller number is lost if it is much less than epsilon
b. the smaller number is lost if its ratio to the larger number is much greater than epsilon
c. the smaller number is lost if its ratio to the larger number is much less than epsilon
d. the smaller number is lost if its ratio to the larger number is much less than underflow
e. the smaller number is lost if its ratio to the larger number is much greater than underflow

5. The fractional error in any float number caused by mantissa roundoff is approximately
a. 2^0 b. 2^{-7} c. 2^{-23} d. 2^{-38} e. 2^{-127}

6. The value of the decimal fraction 11.125_{10} in binary is
a. 1111.1 b. 100.1011 c. 11.111 d. 1011.001 e. 11.001

7. The next float number greater than $1-2^{-23}$ is approximately
a. 1 b. 2^{-22} c. $1-2^{-22}$ d. $1+2^{-22}$ e. $1+2^{-23}$

8 The underflow limit for a `float` number occurs when
a. the exponent in memory equals 1 b. the mantissa in memory equals 0
c. the exponent in memory equals -127 d. the mantissa in memory equals 2^{-23}
e. the exponent in memory equals 2^{-23}

9. The term **fractional** or **relative error** for a series can be defined as
a. (series value – exact value)*exact value b. series value/exact value
c. exact value/(series value - exact value) d. series value – exact value/series value
e. none of the previous

10. Which of the following is correct for the hexadecimal numbers composed from the bit field of a `float` number in memory?
a. the sign bit is combined with the first 3 bits of the exponent
b. the sign bit is combined with the first 3 bits of the mantissa
c. the hidden bit is combined with the first 3 bits of the exponent
d. the hidden bit is combined with the first 3 bits of the mantissa
e. the hidden bit and the sign bit are not stored in memory

11. A condition for a Maclaurin series to be a representation of a function is
a. each term is larger than the one preceding it b. the first term must be 1.0
c. the function and its derivatives must be zero at the origin
d. the highest order term in the series is x^{10} e. none of the previous

12. The second term in the Maclaurin series approximation for xcos(x) is
a. x b. $x^2/2$ c. $-x^2/2$ d. $-x^3/2$ e. none of the previous

13. For a function defined as y = f(x), a Maclaurin series of the function is a series in
a. powers of f(0) b. powers of f(x) c. powers of x d. powers of y e. powers of the derivatives of f(x)

14. What is special about the bit field in memory of the `float` number 0?
a. the exponent is 0 and the mantissa is just the hidden bit b. the mantissa is all 0 and the exponent is 1
c. the mantissa is all 0 and the exponent is 255 d. the exponent is all 0 and the mantissa is all 1
e. none of the previous

15. Which of the following could represent the `float` symbol `NaN`?
a. the exponent is all 0 and the mantissa is all 0 b. the exponent is all 0 and the mantissa is all 1
c. the exponent is all 1 and the mantissa is all 0 d. the exponent is all 1 and the mantissa is all 1
e. none of the previous

16. A truncation error occurs when
a. a computer program terminates b. a double is rounded to a ***float*** c. the program casts a float to an int
d. a number is rounded in the computer e. an infinite mathematical series is terminated

17. Which of the following is the best series to represent the function $f(x) = e^{-x}$ near the point f(x) = 1.0 ?
a. a Maclaurin series b. a Taylor series with a = 1.0 c. a Taylor series with a = 10
d. a Taylor series with a = 100 e. wrong, a series can't be used because the derivatives are zero

18. Which of the following is the third term in the Maclaurin series for $f(x) = e^{2x}$?
a. 2x b. 4x c. x^2 d. $2x^2$ e. $4x^2$

19. Which of the following functions is **NOT** a good candidate for a Maclaurin series?
a. $e^x \sin(x)$ b. sin(x).cos(x) c. e^{2x} d. 1/x e. wrong, they are all good candidates

20. An estimate of the truncation error in a Maclaurin series approximation without knowing the exact value is
a. the last term of the series b. the first truncated term that is zero
c. the first non-zero truncated term d. the sum of all the truncated terms
e. the difference between the exact value and the series value

Short Questions Show all working to get partial marks

21. **[10 marks]**
Show the derivation of the **hexadecimal** field in memory that represents the `float` decimal number -1.25

22. [10 marks]
22a [5 marks]
Derive the Maclaurin series expansion for the function $f(x) = (e^x + e^{-x})/2$ for the first three non-zero terms

22b. [5 marks]
Write a numerical expression for the estimated % fractional (relative) error in your series from **22a** at x = 1.0 when only two terms are used.

Formulas

Maclaurin Series:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + \ldots.$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{for all } x$$

$$\frac{d(\sin(x))}{dx} = \cos(x); \frac{d(\cos(x))}{dx} = -\sin(x); \frac{d(e^{ax})}{dx} = ae^{ax} \qquad \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$