Last Name: ___________________ANSWERS______________________

First Name: ____

Student Number: ____

Instructions:

1. Print your name and student number above AND on the MC answer sheet. A test or MC answer sheet without a name and student number won't be marked. A page (except the front page) without a name at the top right, where shown, won't be marked.
2. Use only a pencil when filling in the MC answer sheet for the multiple choice questions. Circle the correct answers on your question paper first and only when you are certain of your answer fill in the MC answer sheet. Only the answers found on the MC answer sheet will be used when marking the multiple choice questions.
3. Check that you have all 5 PAGES before beginning the exam.
4. Pace yourself – you have ~45 minutes.
5. Use the blank spaces on exam pages for rough work. No scrap paper is permitted.
6. Simple non-programmable calculators (not cell phones or tablets) are allowed – but you can leave your answers as accurate and complete numerical expressions where calculations are required but need not waste time evaluating them.
7. If you have a cell phone or any electronic device (other than a pacemaker) with you – be sure it is turned off now, and stored in a safe place away from your desk.
8. Hand in BOTH this exam booklet AND the MC answer sheet. Taking an exam booklet from the exam room will result in an automatic grade of 'F' for this course.
9. This test is worth 17.5% of your final mark.

I have read, understood, and will comply with all of the above instructions:

____ ___________________

sign your full name here date

Formulas are at the end. Choose the **best** answer in each of the following multiple-choice questions**.**
1 mark each

1. The mantissa of a normalized float number including the hidden bit

a) is approximately 10^{-7} b) holds the power of 2 of the number
<u>c) has a value between 1 and almost 2</u> d) has a value between 0 and almost 1
e) has a value between 0 and 10^{-7}

2. In Science and Engineering real numbers have the following three parts:

a. exponent, characteristic, base b. mantissa, significand, exponent c. base, bias, mantissa
<u>d. mantissa, exponent, base</u> e. exponent, bias, base

3. Which of the following numbers is invalid

a. 2.9979×10^8 b. 0.29979×10^9 c. 29979×10^4 d. 299790000
<u>e. none of the previous are invalid</u>

4. For the float data type the bias of 127 is

a. the amount by which the number is normalized
b. the number that is added to the exponent in memory to give the actual exponent
<u>c. the number that is subtracted from the exponent in memory to give the actual exponent</u>
d. the number that is added to the mantissa in memory to give the actual mantissa
e. the number that is subtracted from the mantissa in memory to give the actual mantissa

5. For the `float` number 128.0 the exponent in memory is

a. 1 b. 120 c. 127 <u>d. 134</u> e. 255

6. What is the binary (base 2) value of the decimal number 13.125?

<u>a. 1101.001</u> b. 111.010 c. 1101.125 d. 111.125 e. .011

7. A de-normalized number has

a. exponent: 0, mantissa: 0 <u>b. exponent: 0, mantissa: not 0</u> c. exponent: not 0, mantissa: 0
d. exponent: not 0, mantissa: not 0 e. none of the previous

8. The symbol number NaN occurs when

a. a double is cast to a float b. a float is cast to a double c. the result of a calculation is 0
d. a float exceeds the value $1.0e^{38}$ <u>e. none of the previous is correct</u>

9. What is true about the float number 0?

a. It doesn't exist – underflow is the closest you can get b. It only has the hidden bit – all the other fields are 0
c. the exponent is 127 (so when the bias is subtracted you get 0)
d. there's nothing special – it's just another normalized number <u>e. none of the previous is true</u>

The following code is used in question 10 - 13

```
float s = 2.0e-8;
float t = 5.0e-7;
float v = 1.0e-4;
float a = 1.0;
float b = 100.0;

printf("%.8e\n", a + t);      // 1.
printf("%.8e\n", a + s);      // 2.
printf("%.8e\n", b + v);      // 3.
printf("%.8e\n", b + t);      // 4.
```

10. Output 1 is likely to be close to

`a. 1.00000000e+000` <u>`b. 1.00000048e+000`</u> `c. 1.00000000e+002` `d. 1.00000099e+002`
`e. none of the previous`

11. Output 2 is likely to be close to

<u>`a. 1.00000000e+000`</u> `b. 1.00000048e+000` `c. 1.00000000e+002` `d. 1.00000099e+002`
`e. none of the previous`

12. Output 3 is likely to be close to
`a. 1.00000000e+000     b. 1.00000048e+000     c. 1.00000000e+002     d. 1.00000099e+002`
`e. none of the previous`

13. Output 4 is likely to be close to
`a. 1.00000000e+000     b. 1.00000048e+000     c. 1.00000000e+002     d. 1.00000099e+002`
`e. none of the previous`

14. A converging Maclaurin series is one where
a. each term is smaller than the preceding term
b. each term is larger than the preceding term
c. each term is a power series in x
d. each term uses derivatives at the origin
e. none of the previous

15. A Maclaurin series does which of the following for a function y = f(x)
a) writes it as a series of terms in powers of y
b) writes it as a series of terms in powers of f(x)
c) writes it as a series of terms in powers of the derivatives of f(x)
d) writes it as a series of terms in powers of x
e) none of the above

16. With reference to the formula at the end, which one of the following is a general expression the third term in the Maclaurin series?
a. $\frac{f^{(3)}(0).x^3}{3!}$ b. $\frac{f^{(3)}(3).x^3}{3!}$ c. $\frac{f^{(2)}(2).x^2}{2!}$ d. $\frac{f^{(0)}(2).x^2}{2!}$ e. $\frac{f^{(2)}(0).x^2}{2!}$

17. The difference between a Maclaurin series and a Taylor series is
a. A Maclaurin series uses derivatives of x and a Taylor series uses powers of x
b. A Maclaurin series uses powers of x and a Taylor series uses powers of the derivatives of x
c. A Maclaurin series uses derivatives at x=0 and a Taylor series uses derivatives at x = a
d. A Maclaurin series uses derivatives at x=a and a Taylor series uses derivatives at x = 0
e. there is no difference between a Maclaurin series and a Taylor series

18. Which one of the following is the most appropriate relative (fractional) error expression of a Maclaurin series for fast executing code?
a. (1st truncated term)/(exact value from math library) b. (1st truncated term)/(Maclaurin series approximation)
c. (all truncated terms)/(Maclaurin series approximation)
d. (all truncated terms)/(exact value from math library)
e. (last truncated term)/(Maclaurin series approximation)

19. Using the Maclaurin series for the cos(x) function in the formulas at the end, what is the slope at x = 0?
a. 0 b. 1 c. $1 - x^2/2!$ d. 1 - x e. 1 + x

20. In the following Maclaurin series for tan(x) near x = 0:

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \ldots$$

what is an estimate of the truncation error if the series has all terms up to and including the term in x^3

a. $x + \frac{1}{3}x^3$ b. $\frac{2}{15}x^5$ c. $\frac{17}{315}x^7$ d. $\frac{17}{315}x^7 + \frac{62}{2835}x^9$
e. there is no error because the series ends at the term in x^3

Short Questions Show all working to get partial marks

21. [10 marks]

Show the derivation of the **hexadecimal** field in memory that represents the `float` decimal number 2.125

$2.125_{10} = 10.001_2$ **->** normalized **=** 1.0001_2 Sign bit = 0
Exponent = 127 + 1 = 128 = 10000000_2 for float
Mantissa = [1] . 0001_2
So the total bit field for 2.125 float = 0100 0000 0000 1000 0000 0000 0000 0000 = 40080000_{16}

22. [10 marks]
22a [5 marks]

Derive the Maclaurin series expansion for the function f(x) = 2cos(x/2) for the first three non-zero terms

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

From the formulas:
Substituting x/2 for x and multiply by 2, for the first three terms:
2cos(x/2) = 2 – 2x²/4.(2!) + 2x⁴/16.(4!) = 2 – x²/4 + x⁴/192

22b. [5 marks]

Write a numerical expression for the estimated % fractional (relative) error in your series from **22a** at x = 1.0 when only two terms are used.
At x = 1.0 with two terms f(x) = 2 – 1/4 = 7/4
and error is ~first truncated term = 1/192
so the % fractional error = 100 x (1/192)/(7/4) = 0.298%

Formulas

The general formula for Maclaurin Series:

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0).x^k}{k!}$$

Maclaurin Series of Particular Functions:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{for all } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

Derivatives:

$$\frac{d(\sin(x))}{dx} = \cos(x); \frac{d(\cos(x))}{dx} = -\sin(x); \frac{d(e^{ax})}{dx} = ae^{ax} \qquad \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$